DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

192. Also solved by J. Scheffer, Hagerstown, Md.

193. Proposed by SAUL EPSTEEN, Ph. D., Chicago, Ill.

Professor Goursat states (*Transactions of the American Mathematical Society*, January, 1904, p. 111) that if a_1, a_2, ..., a_n; h_1, h_2, ..., h_n are two sequences, the h's being all positive, then $\sum \frac{a_i^2}{h_i} \geqq \frac{(\Sigma a_i)^2}{\Sigma h_i}$. Prove this.

Solution by F. L. GRIFFIN, S. B., and L. E. DICKSON, Ph. D., Chicago, Ill.

For $n=2$, the inequality becomes, upon multiplication by the *positive* number $h_1 h_2(h_1+h_2)$ and transposition of terms, $(a_1h_2-a_2h_1)^2 \geqq 0$. These steps may be reversed, giving a proof for $n=2$. For the general case, we proceed by induction, assuming the formula true for $n=1$, 2, ..., m. Then

$$\sum_{i=1}^{m+1} \frac{a_i^2}{h_i} \geqq \frac{\left(\sum\limits_{i=1}^{m} a_i\right)^2}{\sum\limits_{i=1}^{m} h_i} + \frac{a^2{}_{m+1}}{h_{m+1}} \geqq \frac{\left(\sum\limits_{i=1}^{m} a_i + a_{m+1}\right)}{\sum\limits_{i=1}^{m} h_i + h_{m+1}} \text{ or } \frac{\left(\sum\limits_{i=1}^{m+1} a_i\right)^2}{\sum\limits_{i=1}^{m+1} h_i}.$$

Also solved by G. B. M. Zerr, Parsons, W. Va., for sequences where

$$a_i = a_{i-1} + m\,;\ h_i = h_{i-1} + 1.$$

195. Proposed by W. J. GREENSTREET, A. M., Editor of the Mathematical Gazette, Stroud, England.

Prove that when n is a positive integer,

$$\sum_{r=1}^{r=n} (-1)^r\, {}_nC_r\, 2^{n-r}\, r^2 = n^2 - 2n.$$

Solution by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

$$(1+x)(1-x)^{-3} = 1+2^2x+3^2x+3^2x^2+4^2x^3+\ldots\ldots(2x-1)^n$$
$$= 2^nx^n - c_1 2^{n-1}x^{n-1} + c_2 2^{n-2}x^{n-2} - \ldots\ldots$$

Required sum=coefficient of x^{n-1} in $(1+x)(1-x)^{-3}(2x-1)^n$,

i. e., in $(1+x)(1-x)^{-3}[x^n - c_1x^{n-1}(1-x) + \ldots\ldots]$,

i. e., in $(1+x)[x^n(1-x)^{-3} - c_1x^{n-1}(1-x)^{-2} + \ldots\ldots]$,

which is $2c_2 - c_1$ or $n^2 - 2n$.

Also solved by G. B. M. Zerr, A. M., Ph. D., Parsons, W. Va.